ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-IV

Paper Code: C8T

[Riemann Integration and Series of Function]

Answer all the questions

1. Let a function $f:[a,b] \to \mathbb{R}$ be bounded on [a, b] and let f be continuous on [a, b] except for a finite number of points in [a, b]. Then f is Integrable on [a, b].

2. Define uniform convergent of a sequence. A sequence of function $\{f_n\}$ is defined by $f_n(x) = \frac{nx}{1+n^2x^2}$, $0 \le x \le 1$.Show that the sequence $\{f_n\}$ is not uniformly convergent on [0, 1].

3. A function f is defined on [0, 1] by f(x)=$\sin x$, if x is rational and f(x)=x , if x is irrational (i) Evaluate upper and lower integral of f on $[0, \frac{\pi}{2}]$. (ii) Show that f is not integrable on $[0, \frac{\pi}{2}]$.

4. Let $f(x) = x[x]$,$x \epsilon[0\ ,3]$. Show that f is Integrable on [0, 3]. Evaluate $\int_0^3 f$.

5. A function f is continuous on $\mathbb{R}$ and $\int_{-x}^{x} f(t) = 0$ for all x ϵ $\mathbb{R}$ .Prove that f is an odd function on $\mathbb{R}$.

6. Examine the convergence of the improper integrals.

   (i) $\int_0^1 \frac{1}{\sqrt{x(1-x)}} dx$ (ii) $\int_0^1 logx$ dx

7. let $D \subset \mathbb{R}$ n$\in$ $\mathbb{N}$,$f_n : D \to \mathbb{R}$ is continue on D then show that the uniform converges of the sequence $\{f_n\}$ on D is a sufficient but not a necessary condition for continuity of the limit function on D .

8. Let $\sum a_n x^n$ be a power series with radius of $onvergenceR(> 0)$ .Let f(x) be sum of the power series on (-R,R) .Then prove that f(x) is continuous on (-R,R) .

9. If f(x) be the sum of the series e^{-x}+2$e^{-2x} + 3e^{-3x}$+………,x >0 .$show$ that f(x) is continuous on (-R,R) .

   $$Evalute \int_{log_e^2}^{log_e^3} f(x)dx$$

10. State and prove $cauchys$ criteria for uniform convergence of the sequence of functions .

11. For the series $\sum_{n=1}^{\infty} f_{n(x)}$ where f_n=n^2x$e^{-n^2x^2}$ – $(n-1)^2$x $e^{-(n-1)^2x^2}$, x$\in$ [0,1]. Show that

12. $\sum_1^{\infty} \int_0^1 f_{n(x)dx} \neq \int_0^1 (\sum_1^{\infty} f_n\,(x)\,)\ dx$. Is the series $\sum_1^{\infty} f_{n(x)}$ uniformaly convergent on [0,1] ?

13. Show that the sequence of functions { f_n(x) } where f_n (x) =$\frac{nx}{1+n^2x^2}$, ∀ x∈ ℝ is not $uniformaly$convergent in any interval [a b] which contains 0

14. let D ⊂ ℝ and let { f_n]] be a sequence of $functionspointwise$ convergent to f(x) .Let

$M_n = \underset{x \in D}{SUP} |f_n(x) - f(x)|$. Then

$showthat\ \{f_n\}\ isuniformalyconvergentonD$ to f(x) if and only if $\lim_{n\to\infty} M_n$= 0 .

15. Assuming that the sum of the power series x- $\frac{x^2}{2}$ +$\frac{x^3}{3}$ ….. on its interval of convergence is $\log_e(1+x)$ deduce that

$\frac{1}{1.2}$ - $\frac{1}{2.3}$ + $\frac{1}{3.4}$ -……..= $2log2$– 1

16. Let f: [-π ,π] → ℝ be continuous except for at most a finite number of jumps is periodic of period 2π , then prove that

$\frac{{a_0}^2}{2} + \sum_{k=1}^{n}(a_k^2 + b_k^2\ ) \leq \frac{1}{\pi}\int_{-\pi}^{\pi} f^2$ (x) dx where $a_k and\, b_k$ are the$fourier$ coefficient of f(x) defined by

$a_k\ = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)cosntdt\ , n \geq 0$

$b_k\ = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)sinntdt\ ,\ \ forn \geq\ 1$

17. Define a uniform continuity of a function on interval [a, b] show f(x) = $\frac{1}{x}$,x ≠ 0 is not uniformaly continuous in (0,1) .

18. Using Cauchy s principle prove that $\lim_{x\to 0} cos\frac{1}{x}$ does not exist .

19. Find $\overline{\lim} u_n$ and $\underline{\lim} u_n$ where $u_n = n + \frac{(-1)^n}{n}$

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